$$\text{and } \angle LCI=(90^\circ-\angle LCD)=90^\circ-2\tan^{-1}\sqrt{\frac{(R-a-b)(b-c)}{c(R-a)}}.$$

$$\therefore \text{ Area of sector } LDT=\frac{360^\circ}{90^\circ-2\tan^{-1}\sqrt{\frac{c(R-a-b)}{(R-a)(b-c)}}}[\pi(R-a-c)^2]\ldots\ldots(5),$$

$$\text{and area of sector } LCI=\frac{360^\circ}{90^\circ-2\tan^{-1}\sqrt{\frac{(R-a-b)(b-c)}{c(R-a)}}}[\pi(R-a-b+c)^2]..(6).$$

Summing (1), (2), (3), (4), (5), and (6), we get the area grazed over,

$$\tfrac{1}{4}\pi(4R^2+b^2+2c^2-2bR-2bc)+\sqrt{[c(R-a)(R-a-b)(b-c)]}$$

$$+\frac{360^\circ}{90^\circ-\tan^{-1}\sqrt{\frac{c(R-a-b)}{(R-a)(b-c)}}}[\pi(R-a-c)^2]$$

$$+\frac{360^\circ}{90^\circ-2\tan^{-1}\sqrt{\frac{(R-a-b)(b-c)}{c(R-a)}}}[\pi(R-a-b+c)^2].$$

Solutions of problem 106 were received from P. S. Berg and Elmer Schuyler, and of problem 105 from Sylvester Robins. These solutions came too late for credit in last issue.

NOTE ON THE CALCULATION OF INTEREST AND DISCOUNT.

BY JOSEPH V. COLLINS, PH. D., PROFESSOR OF MATHEMATICS, STATE NORMAL SCHOOL, STEVENS POINT, WIS.

In the January number of the MONTHLY, Hon. J. H. Drummond, after giving the answer to the problem, 'What are the proceeds of a note discounted at a bank for 10 years at 10 per cent.,' as *nothing*, says, "The method of calculating discount used by banks was invented to evade the usury laws. I have thought that the court which first sustained the method could not have been well versed in mathematical principles."

A curious thing and commentary on the preceding is that all methods of calculating interest and discount are open to objection. Custom requires that interest be paid at the end of each year or specified fraction of a year. When it is not paid until the end of a period of two or more years, the lender is defrauded of the interest on his interest. If all interest payments are made promptly, it is equivalent to paying compound interest. The business world recognizes that compound interest is the only fair kind. Thus tables of bond values and the like are always made on a compound interest basis. But simple interest is

the only kind which can be collected unless annual or compound is called for in the contract. The courts have decided, probably because compound interest piles up so rapidly, that borrowers shall not be compelled to pay it unless they have agreed to. It is evident that if simple interest is unfair to a lender, so likewise is its analogue in discount, viz., true discount. In bank discount the bank not only collects a certain rate of interest on the loan, but also *on its own pay*, which is theoretically an absurd proceeding.

In partial payments we find the same difficulties presenting themselves. If payments are made within a year, according to the United States rule interest is collected and set at interest before it is due. The old so-called Connecticut rule tried to avoid this unfairness, but in so doing became too complicated for general use. Business men see that the mercantile rule is the only one which is fair to both parties when the whole transactions falls within a year. But the mercantile rule works injustice if the period covers more than a year, since by it the lender gets only simple interest, whereas the United States rule gives him a form of compound interest. The United States rule works injustice to the borrower whenever he pays less than the interest. Thus it is evident that the element of time and certain practical considerations have a great deal to do towards determining the appropriate method of counting interest.

Looked at from a practical standpoint it is easy to see why the banker collects bank instead of true discount. True discount requires a long division after a preliminary calculation. Tables could not be made for computing true discount which would be at all convenient to use. The great bulk of the loans made by banks are for less than 3 mouths. The difference between the bank and true discount of say $500 for 30 days is only a little over one cent. It would be worth more than 5 cents of the cashier's time to make the longer calculation. Of course the difference would not be so slight in every case, but it should be noted that one and sometimes more than one other person's time besides the cashier's is involved. Then liability to error is much greater in the longer calculation, and this is an important item. Hence it is plain that the method of discounting notes pursued by the banks was not adopted (as the writer once thought before he began making computations like the above) with the object of extorting more money from their customers, but for purely practical reasons.

ALGEBRA.

92. Proposed by W. F. BRADBURY, A. M., Head Master Latin School, Cambridge, Mass.

Find the sum to n terms of $1+3^3+5^3+\dots$ [From Charles Smith's *Elementary Algebra*, page 403].

I. Solution by DR. E. D. ROE, Jr., Associate Professor of Mathematics in Oberlin College, P. O., Norwood, Mass.

We have $s_{3,\,n}=1^3+2^3+\dots n^3=\left(\dfrac{n\,(n+1)}{2}\right)^2$. (Todhunter's *Algebra*, page 263).